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Sampling the QUALITY of Hardwood Trees

by Adrian M. Gilbert

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The Question of Quality

ANYONE acquainted with the conversion of hardwood trees into wood products knows that timber has a wide range in quality. Some trees will yield better products than others. So, in addition to rate of growth and size, tree values are affected by the quality of products yielded.

To convert a hardwood tree into wood products, the stem is cut into sections, which are classed by intended use. These use classes include sawlogs, veneer logs, structural logs, boltwood, and pulpwood. They vary in value. Some classes have such a wide range of value that they may be divided into grades. Now, if there were specifications for all these classes and grades, it might be possible to estimate the conversion value of the stem sections.

Figure 1.--*Old-growth sugar maple trees on the Bartlett Experimental Forest vary considerably in quality.*

FIRST, LOG GRADING

Since more than half of all hardwoods cut are converted into factory lumber, a start was made with sawlogs. In 1949, the Forest Products Laboratory of the U.S. Forest Service proposed its Hardwood Log Grades for Standard Lumber (6). These are by no means the only hardwood log grades in existence, but the Laboratory grades have been adopted by the Forest Service. And they have become a yardstick by which other grades may be compared.

In grading factory logs, measurements of size, straightness, sound and unsound end defects, and measurements of surface areas free from bark features that indicate degrading blemishes in the wood are combined in specifications for log grades. Each grade has distinct and predictable factory lumber yields. In a similar way, yields of structural timbers and pulpwood can be predicted from other stem sections.

NOW TREE GRADING

If the output of products from logs can be estimated, can't the same thing be done with trees? Yes, it appears that it can; and a search is being made for practical tree grades. Numerous systems have been suggested and are being tested in many parts of the hardwood region.

In most stands, the trees have a wide range of characteristics that affect type and value of output. Some of the factors affecting product-output values are not as easy to evaluate in the standing tree as in the log. This is especially true of rot, shake, stain, and end defects. It also requires skill to identify and evaluate some other defects, such as overgrowths and low bumps. In applying log grades directly to trees, variation in bucking practices can affect values significantly. But with training and experience, these difficulties can be minimized, and the grading of logs in trees can be accomplished.

Assuming that practical tree grades can be developed, one question always arises, "How many trees should be graded to estimate the total value of a stand or forest?"

A Study of Tree Quality

A study of tree quality made in New England led to development of a method for determining the size of the sample that should be graded to get an estimate of the value of a forest. This method was field-tested on the Bartlett Experimental Forest at Bartlett, New Hampshire.

THE NEED FOR SAMPLING

On small areas, such as sample plots, it would be as practical to grade as to tally all trees. But, on large areas, every tree need not be counted, and every tree need not be graded to obtain a satisfactory estimate at an acceptable cost. The number of sample trees to be measured for volume determination can be calculated. The variations leading to their determination are understood and the procedures are well established. Less is known about the factors affecting reliability of value estimates.

The size of sample needed to estimate total value depends upon: (1) The desired accuracy of total value, (2) the variation in quality (value) among individual trees, and (3) the availability of a tree-grade system that permits use of stratified sampling.

In this study the variation in quality between individual trees was considered on the assumption that, if log-end features were ignored and reasonable allowance were made for bucking variations, the available hardwood log-grading information could be applied accurately enough so that reliable estimates of gross product-output value could be made.

STUDY MATERIAL

The grades of 250 sugar maple (Acer saccharum) trees, 11.0 inches d.b.h. and larger were tallied by sections (3) These were a sample of old-growth northern hardwoods stands on the Bartlett Experimental Forest (fig. 1).

Briefly, there were three phases to the study: (1) computing the conversion values of the trees, (2) analyzing the variation in values, and (3) calculating sample size.

COMPUTING CONVERSION VALUES

Section Grades

In the grading system used, the stem was divided into 16-foot sections comparable to logs, starting from the stump (table 1). To compensate for possible upgrading by bucking into shorter lengths, the grade of the best 12 or 14 feet of the section was assigned to the entire section (7). Top logs 12 feet and longer were graded on the basis of actual length and were admitted to all grades. Top logs less than 12 feet long were graded as half logs and were admitted only to grades 3, 4, and 5.

When a section was considered for factory grades, it was visually squared into four faces (fig. 2). The grade of the section was the grade of the second poorest face.

Table 1.--Proposed hardwood grades for 16-foot sections of standing trees[1]

Section grading factors	Grade 1‡			Grade 2‡	Grade 3‡	Grade 4†	Grade 5*
Top d.i.b.	20+	16-19	[2]13-15	[3]11	8	8	8
Clear cuttings:							
Length	3	5	7	3	2	--	--
Number	2	2	2	3	--	--	--
Yield	5/6	5/6	5/6	2/3	1/2	--	--
Sweep	15%	15%	15%	30%	50%	1/2 dib 16' log	1/2 dib 8' log
Cull and sweep	40%	40%	40%	50%	50%	([4])	[5]50%

[1] Top logs less than 12 feet long cannot make grades 1 and 2.

[2] Ash and basswood butts can be 12 inches if otherwise grade 1.

[3] 10-inch logs of all species can be grade 2 if otherwise grade 1.

[4] Knot diameter less than 1/3 log diameter at point of occurrence; no interior unsound defects allowed.

[5] Knot diameter less than 1/2 log diameter at point of occurrence.

‡Adapted from specifications for Forest Service Standard Grades for factory lumber logs.

†Adapted from specifications for Forest Service Standard Grades for construction lumber logs.

*Adapted from Forest Survey specifications for local-use logs.

Actually, the section grades 1, 2, 3, in table 1 were the same as Forest Service Standard log grades for 12-foot and longer factory logs, except that sound end defects were not considered. Each section of the stem was graded into one of the three factory grades, a structural grade, a local-use grade, or cull (fig. 3).

Product Yields

Having adopted a grading system, the next step was to determine product yields by these grades. The lumber-grade yields for factory class logs were computed from Forest Products Laboratory data for such logs (6). For each species the log grade and the lumber grade yields in percent are given by log d.i.b. For log grades 1 and 2 of sugar maple, the grade yields for FAS lumber were smoothed by free-hand curves to reduce apparent inconsistencies between diameter classes (Table 2, Appendix).

For use classes other than factory lumber, it was assumed that the yields of the sections would be completely of that class: for example, that structural sections (Grade 4) would yield 100 percent ties and timbers; local-use sections (Grade 5) would yield 100 percent pulpwood. These assumptions were necessary for purposes of analysis. Though they do not always hold in actual practice, they had no significant effect upon the results of the study.

Quality Indices

Now, product yields in percent of product grades or in dollars are helpful. However, they can be handled more easily when expressed as an index. The concept of an index for the quality of hardwood logs was developed by Herrick in 1945 (4). Herrick found that the ratios of reported prices of lumber grades are very stable (5). For example, the ratio of the reported price of sugar maple FAS lumber to No. 1 Common has remained almost constant for the past 25 years. Since the price relations remain constant over a period of time, the variations in quality expressed by the quality index are for all practical purposes valid.

For a species, or group of species, the prices of various grades of lumber were expressed as ratios of No. 1 Common, which was taken as unity. As many as five different grades of

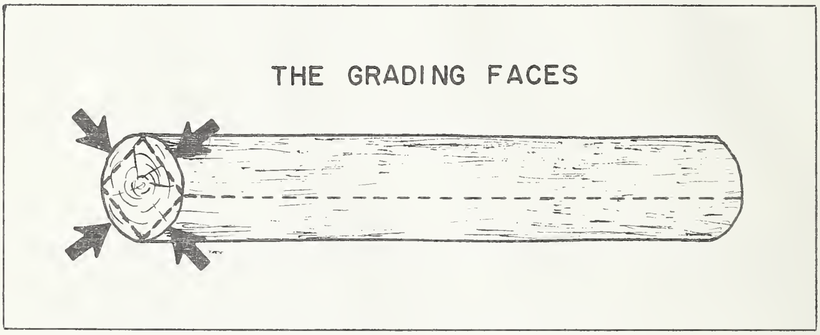


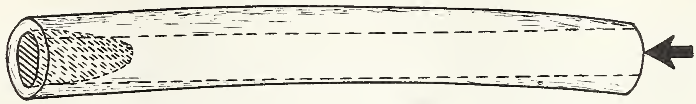
Figure 2.--*For judging quality, a tree section is visually squared into four faces, and each face is judged as though it were a board. Tree sections are graded on a 16-foot basis, and the grade of the best 12 or 14 feet is assigned as the grade of the entire section.*

lumber may be sawed from a hardwood log. To find the Quality Index (Q.I.) of the log, the percentage of each lumber grade sawed was multiplied by the price ratio of the grade. The sum of the products is the Q.I. of the log. The dollar value of lumber a log will yield is given by the product of Q.I., the price of No. 1 Common lumber, and log volume.

For structural or local-use classes, the Q.I. of the log was the ratio of the price of ties or pulpwood to No. 1 Common lumber. Dollar values would be computed as for factory-grade logs.

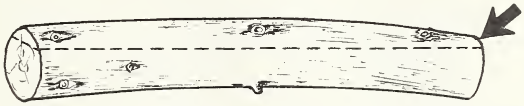
The quality index of a tree can be computed in a similar manner. The Q.I. of a tree is the weighted sum of the indices of the logs in the tree. This is a very useful number.

A GRADE 1 FACTORY LOG



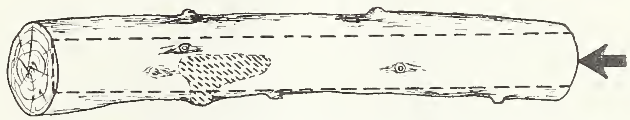
A 16-foot log 20 inches in diameter at the small end. Less than 15 percent deduction for sweep. Total deduction for sweep and rot is less than 40 percent. There are no surface indications of defect.

A GRADE 2 FACTORY LOG



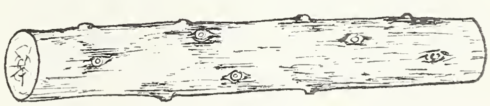
A 12-foot log 18 inches in diameter at the small end. More than 2/3 of its grading-face length is clear in two sections 5 and 4 feet long. Deduction for sweep is about 20 percent. Total deduction is less than 50 percent.

A GRADE 3 FACTORY LOG



A 14-foot log 22 inches in diameter at the small end. More than ½ of its grading-face length is clear in three sections 2, 3, and 4 feet long. Less than 50 percent deduction for sweep and rot.

A STRUCTURAL GRADE LOG



A 12-foot log 18 inches in diameter at the small end. It has numerous knots, but none with a knot collar exceeding 1/3 of the log diameter at the point where it occurs. No unsound defect. The log is straight, so there is no problem of sweep.

Figure 3.--*Examples of grading for quality. The second poorest face (indicated by arrow) was used to determine the grade.*

It is a yardstick by which tree values can be compared. And, it provides a suitable base for sampling the conversion quality of trees.

Prices.--Product prices for this study came from these sources: factory lumber prices from the Commercial Bulletin, Boston; structural lumber from tie prices of a New England railroad; local-use prices from the roadside price of pulpwood at the Bartlett Experimental Forest.

Product	Actual price[1]	Price ratio
Factory lumber		
FAS	185	1.7
Select	165	1.5
No. 1 C	110	1.0
No. 2 C	65	.6
No. 3	45	.4
Structural	55	.5
Local use	35	.3

There is no historical basis for assuming that the ratio of structural and pulpwood prices to the price of No. 1 Common lumber has been stable. But this will not affect the practical use of Q.I. as a sampling yardstick. The same is true of Q.I.'s developed from prices in other markets.

Taper tables.--The next step was to get a taper table for estimating the d.i.b. of the small end of 16-foot sections by diameter classes and tree lengths. The Forest Survey gave us one that is used as an average for hardwoods (Table 3, Appendix). Use of individual tree form class to obtain these data was too great a refinement for this study.

Quality indices.--An intermediate table of quality indices by log grades and d.i.b. was prepared, using the appropriate price ratio and product-yield data. This table was converted to a master table of quality indices by position of the section in the tree through the use of the taper table (Table 4, Appendix). These data were then used to obtain a quality index for each tree, weighting sectional indices by their gross cubic-foot volume.

VARIATION OF TREE QUALITY INDICES

First analysis of the data showed such a wide variation in Q.I. that it was obvious an impracticably large sample would have been required to get an estimate of total value within the stipulated accuracy limits. Therefore, a test was made of

[1]Lumber prices are average weekly prices f.o.b. mill for 1953-1954.

stratifying the tree data to see if this device would lead to a sample of practical size.

To determine the suitability of stratification, an uncommon twist was given to the analysis of variance (8). Usually, the variance ratio is mean square between classes divided by mean square within classes. Here, the test of significance was a little different. The variance ratio used was total mean square divided by mean square within classes. The ratio is an estimate of how many more samples would be needed with unstratified sampling as compared to stratified sampling. If the ratio were greater than one and significant, there would be strong case for stratified sampling.

Table 5 (Appendix) shows the results of the analysis of variance, testing several methods of stratification: (1) by diameter class, (2) by butt log class within diameter classes, (3) by butt log class, (4) by diameter classes within butt log classes. Actually, stratification by d.b.h. class or by butt log grade within d.b.h. class are the best possibilities since we usually measure tree diameter to get volume estimates. The analysis showed that diameter class alone is the most efficient means of stratification. Thus, a significant and reasonable sized sample can be taken.

Calculating Sample Size

The analysis showed that d.b.h. stratification had a significant advantage over no stratification, and the process appeared practical for application. However, there were differences in quality variation, as measured by Q.I., among diameter classes (2 inch). Variation appeared to increase as diameter increased. Therefore, numbers of samples must be determined for each diameter class. A good way of handling this is to relate the degree of variation in each class to that of the class having the largest deviation.

The procedure, called stratification with variable sampling fraction, was adapted from Yates (8), with the help of C. Allen Bickford, statistician for the Northeastern Forest Experiment Station. An example of the procedure is shown in the Appendix. The area to be sampled was a 48-acre compartment on the Bartlett Experimental Forest, stocked with old-growth northern hardwoods.

To calculate the sample size, two kinds of information were needed:

1. An estimate of tree quality index and its variance for each diameter class. These estimates may come from a trial run or from previous data collected on a similar area. In this case, they came from the 250 trees.

2. An estimate of the numbers of trees in each diameter class of the area to be sampled. Here, the estimate may be from sample plots, from stand tables of similar areas, or guessed. The estimates in the example came from sample-plot measurements.

In addition, someone must decide upon an allowable sampling error. For sugar maple, in the example, we used 10 percent of the total quality index weighted by volume.

The calculations show that for each diameter class about five sample trees are needed (Table 6, Appendix). The sampling intervals (fractions) varied depending upon the number of trees in each class. For ease of handling in the field, the same interval could be used for several classes. In the 12-, 14-, and 16-inch diameter classes, the interval might be 1 in 25. Since larger trees are usually more valuable, every tree above a given diameter, perhaps 20 inches, might be graded. (Also, all sawtimber trees of valuable species that occur infrequently, such as white ash, would be graded. There is a limit to the uses of sampling.)

Now, for each sample tree, every 16-foot section was to be graded, just as the original 250 trees.

One question that arose was: "Over how wide an area is the sample applicable?" The sample can be used for any similar area that has no more variation in quality than the original. This can be checked by a t-test with the null hypothesis that there is no difference in quality between trees in another area and those in the original sample.

For example, a random sample of 21 sugar maple trees was taken from another compartment (No. 34) at Bartlett. These were graded by 16-foot sections and their Q.I.'s were computed. These indices were compared with the average Q.I.'s by d.b.h. classes of the original samples (Table 7, Appendix).

When all the 21 maples were compared, t was too large to be random. However, we expected to grade every sugar maple larger than 20 inches. Then, for the smaller trees, t was small enough to be random. In other words, the average quality indices of sugar maple could be applied to the trees in Compartment 34 that were 20 inches d.b.h. or smaller.

Discussion

LIMITATIONS

The method proposed here of sampling to estimate the conversion value of standing timber is not limited to systems of grading trees by sections. Eventually, more refined tree-grading systems will be developed. But, if we wish to sample standing timber for value as well as volume, we need some way of determining the value of an individual tree.

In this study, quality index (Q.I.) was used for value. Tree Q.I. was constructed from section Q.I., which is a lengthy process. It is based upon assumptions that have not been checked and includes possible measurement and reporting errors. However, we believe the data are accurate enough so that the method proposed is sound.

Actually, an entire stand or forest would be sampled for quality, not just one species, as described in this paper. Then, each species or group of species would be sampled to an accuracy proportional to its contribution to the total value.

As presented here, the practicing forester can make very limited use of this method to determine the size of sample. If there were tables, similar to volume tables, of average quality index (weighted by volume) by species and diameter class, the forester could test a small random sample of trees from his forest as shown in Appendix D. If the differences were not significant, he could use the tables to estimate standing timber values. However, if the differences were significant, other tables would be needed.

ADDITIONAL RESEARCH NEEDED

This study does indicate the need for further research. We need more refined tree grades. We need to know more about measurement and reporting errors. We need to know more about the quality index concept, especially whether it can be extended to products other than sawlogs. Otherwise, new concepts must be developed.

Summary

A tree-quality sampling method is proposed, for use in determining the number of trees to sample to estimate the total value of growing stock.

A system of grading every section in a tree by modified

Forest Service standard grades was used. A method of finding tree quality index is described.

Analysis of variance for 250 sugar maple trees showed a significant advantage in stratifying weighted quality index by d.b.h. classes.

A procedure for calculating sample size is explained and its limitations are discussed. Further study is suggested.

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APPENDIX

COMPUTING CONVERSION VALUES

Table 2.--Lumber grade recovery of sugar maple by log grade and d.i.b., in percent

(Basis: Graphs smoothed from Forest Prod. Lab. Rpt. D1737)

Log and lumber grade	Log diameter inside bark, in inches													
	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Grade 1														
FAS	--	--	13	16	19	21	24	27	29	32	35	38	40	43
Select	--	--	13	13	13	13	13	13	13	13	13	13	13	13
No. 1 C	--	--	31	31	31	31	31	31	31	31	31	31	31	31
No. 2 C	--	--	12	12	12	12	12	12	12	12	12	12	12	12
No. 3	--	--	31	28	25	23	20	17	15	12	9	6	4	1
Grade 2														
FAS	2	3	4	6	7	8	9	10	11	12	14	15	16	17
Select	6	6	6	6	6	6	6	6	6	6	6	6	6	6
No. 1 C	30	30	30	30	30	30	30	30	30	30	30	30	30	30
No. 2 C	21	21	21	21	21	21	21	21	21	21	21	21	21	21
No. 3	41	40	39	37	36	35	34	33	32	31	29	28	27	26
Grade 3														
No. 1 C	15	15	15	15	15	15	15	15	15	15	15	15	15	15
No. 2 C	24	24	24	24	24	24	24	24	24	24	24	24	24	24
No. 3	61	61	61	61	61	61	61	61	61	61	61	61	61	61

Table 3.--Estimated d.i.b. of sections by diameter classes, in inches

D.b.h. (inches)	Position in tree						
	Butt of 1-log tree	Top half of $1\frac{1}{2}$-log tree	Top log of 2-log tree	2nd log of $2\frac{1}{2}$-log tree	Top log of $2\frac{1}{2}$-log tree	2nd log of 3-log tree	Top log of 3-log tree
10	9	--	--	--	--	--	--
12	10	9	--	--	--	--	--
14	11	10	9	--	--	--	--
16	13	12	11	10	9	--	--
18	14	13	12	11	10	9	--
20	16	15	14	13	12	11	10
22	18	17	16	15	14	13	12
24	19	18	17	16	15	14	13
26	21	20	19	18	17	16	15
28	22	21	20	19	18	17	16
30	24	23	22	21	20	19	18

Table 4.--Master table of unweighted quality indices of sugar maple, by position of section in tree

D.b.h.[1] (inches)	Position in tree[1]						
	Butt of 1-log tree	Top half of 1½-log tree	Top log of 2-log tree	2nd log of 2½-log tree	Top log of 2½-log tree	2nd log of 3-log tree	Top log of 3-log tree
				TREE GRADE 1			
14	0.84	--	--	--	--	--	--
16	.92	0.88	0.84	--	--	--	--
18	.96	.92	.88	0.84	--	--	--
20	1.03	1.00	.96	.92	0.88	0.84	--
22	1.10	1.06	1.03	1.00	.96	.92	0.88
24	1.13	1.10	1.06	1.03	1.00	.96	.92
26	1.21	1.17	1.13	1.10	1.06	1.03	1.00
28	1.25	1.21	1.17	1.13	1.10	1.06	1.03
30	1.31	1.27	1.25	1.21	1.17	1.13	1.10
				TREE GRADE 2			
10	0.68	--	--	--	--	--	--
12	.70	0.68	--	--	--	--	--
14	.71	.70	0.68	--	--	--	--
16	.74	.73	.71	0.70	0.68	--	--
18	.77	.74	.73	.71	.70	0.68	--
20	.79	.78	.77	.74	.73	.71	0.70
22	.82	.80	.79	.78	.77	.74	.73
24	.83	.82	.80	.79	.78	.77	.74
26	.87	.84	.83	.82	.80	.79	.78
28	.88	.87	.84	.83	.82	.80	.79
30	.91	.90	.88	.87	.84	.83	.82

[1]For the following tree grades, a single unweighted quality index was used for all diameters and sections:

Tree grade 3 - 0.54
Tree grade 4 - .50
Tree grade 5 - .30

Table 5.--Analysis of variance of sugar maple quality

Source	Degrees of freedom	Sums of squares	Mean square	Variance
Between d.b.h. classes	9	4,669,527.698	518,836.41	
Within d.b.h. classes				
Between butt log classes	21	265,480.299	12,641.92	
Within butt log classes	219	1,278,811.603	5,839.32	$S_1{}^2$
Total within d.b.h. classes	240	1,544,291.902	6,434.55	$S_2{}^2$
Between butt log classes	4	2,944,918.321	736,299.58	
Within butt log classes				
Between d.b.h. classes	26	1,990,089.676	76,541.91	
Within d.b.h. classes	219	1,278,811.603	5,839.32	$S_3{}^2$
Total within butt log classes	245	3,268,901.279	13,342.45	$S_4{}^2$
Total	249	6,213,819.600	24,955.10	S^2

$\frac{S^2}{S_2{}^2} = 3.88$ significant at 0.01

$\frac{S^2}{S_4{}^2} = 1.87$ significant at 0.01

$\frac{S_2{}^2}{S_1{}^2} = 1.10$ nonsignificant

$\frac{S_4{}^2}{S_3{}^2} = 2.28$ significant at 0.01

Table 6.--Calculation of sample size to obtain quality index of sugar maple to specified accuracy for a 48-acre compartment on the Bartlett Experimental Forest

D.b.h. (inches) i	Number of trees N_i	Volume V_i	Weighted Q.I. $\frac{Q_iV_i}{N_i}$	Variance of $Q_i V_i$ S_i^2	Trial fraction* $\frac{S_iF_j}{S_j} = F_i$	Estimated No. of sugar maple on 48 acres M_i	$\frac{M_iQ_iV_i}{N_i}$	$\frac{\frac{M_iQ_iV_i}{N_i}}{\sum\left(\frac{MQV}{N}\right)} = h_i$	$S_i^2h_i$	$\frac{S_i^2h_i}{F_i}$	m_i	No. of samples n_i	Sampling fraction f_i
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
	From preliminary sample												
12	45	6,606	78	340	0.0413	115	8,970	0.086	29	702	4.750	3	1/35
14	33	7,952	146	672	.0580	107	15,622	.151	101	1,741	6.206	4	1/27
16	21	6,814	205	1,405	.0839	52	10,660	.103	145	1,728	4.363	3	1/17
18	84	33,994	276	5,298	.1629	45	12,420	.120	636	3,904	7.330	5	1/9
20	44	22,596	386	9,313	.2159	37	14,282	.138	1,285	5,952	7.988	6	1/6
22	11	6,344	492	13,057	.2557	32	15,744	.152	1,985	7,763	7.222	5	1/6
24	4	2,862	‡562	51,542	.5080	22	12,364	.119	6,133	12,073	11.176	8	1/3
26	2	1,608	632	‡ 91,960	.6786	6	3,792	.037	3,403	5,015	4.072	3	1/2
28	4	4,227	637	146,000	.8550	9	5,733	.055	8,030	9,392	7.695	5	1/2
30	2	2,331	683	199,712	1.0000	3	2,049	.020	--	--	3.000	3	1/1
32	--	--	†700	†225,000	1.0000	2	1,400	.013	--	--	2.000	2	1/1
34	--	--	†700	†250,000	1.0000	1	700	.006	--	--	1.000	1	1/1
Total	250	--	--	--	--	431	103,736	1.000	21,747	48,270	--	48	--

$$e^2 10\% = \frac{\left[10 \frac{(103,736)}{100}\right]^2}{431} = 249,679$$

$$c = \frac{48,270 \quad (t = 2)^2}{249,679 + 21,747} = 0.711354$$

$$m_i = M_iF_i; \quad n_i = cm_i; \quad f_i = \frac{n_i}{M_i} = cF_i$$

*F_j = 1 at 30-inch d.b.h. class; $S_j = \sqrt{199,712} = 466.891$.

‡Adjusted to be in line with adjacent values.

†Guessed values.

Table 7.--Comparison of the quality indices of a random sample of sugar maple with the averages (by diameter class) of the 250 trees in the preliminary data

Random sample			Preliminary sample-weighted Q.I. = X_2	Difference in Q.I. = X	Total difference = ΣX	Mean $\bar{X}$
Tree number	D.b.h. (inches)	Weighted Q.I. = X_1				
1	14	170	146	24		
2	14	146	146	0		
3	14	189	146	43		
4	14	145	146	-1		
5	16	264	205	59		
6	16	200	205	-5		
7	18	391	276	115		
8	18	222	276	-54		
9	18	260	276	-16		
10	18	265	276	-11		
11	20	376	386	-10		
12	20	508	386	122		
13	20	327	386	-59		
14	20	394	386	8		
15	22	608	492	116	215	15.36
16	22	560	492	68		
17	22	560	492	68		
18	22	608	492	116		
19	24	798	470	328		
20	26	919	632	287		
21	30	1303	683	620		
					1818	86.57

$\Sigma_{21}\ (X_1 - X_2)^2 = 651,492$

$s^2_{\bar{X}21} = 1176.44$

$s_{\bar{X}_{21}} = 34.30; \quad t = \frac{86.57}{34.30} = 2.52$ d.f. 20, $t_{0.5} = 2.086$--too large to be random

$\Sigma_{14}\ (X_1 - X_2)^2 = 40,979$

$s^2_{\bar{X}_{14}} = 207.02$

$s_{\bar{X}_{14}} = 14.39; \quad t = \frac{15.36}{14.39} = 1.07;$ d.f. 13, $t_{0.5} = 2.16$--small enough to be random